Section 3.1

Definition of Exponential Function

The exponential function f with base a is denoted by
$$f(x) = a^x$$
where $a > 0, a \neq 1$, and x is any real number.

The Natural Base e

The number $e \approx 2.718281828 \dots$ is called the natural base. The function given by $f(x) = e^x$ is called the natural exponential function.

Formulas for Compound Interest

After t years, the balance A in an account with principle P and annual interest rate r (in decimal form) is given by the following formulas.

a) For n compoundings per year: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

b) For continuous compounding: $A = Pe^{rt}$

Problem 1. Use the graph of f to describe the transformation that yields the graph of g.

a) $f(x) = 3^x$, $g(x) = 3^x - 2$

b) $f(x) = 8^x$, $g(x) = 8^{-x+5}$

Problem 2. Sketch the graph of the following functions: $f(x) = 0.5^x$, $g(x) = e^x$, $h(x) = e^{-x}$.

Problem 3. Solve the following equations for x.

a) $3^{x+1} = 27$

b) $\left(\frac{1}{2}\right)^{x-1} = 64$

c) $e^{5x+4} = e^7$

d) $e^{x^2-24} = e^{2x}$

Problem 4. A total of $12,000 is invested at an annual interest rate of 6%. Find the balance after 8 years if it is compounded

    a) Monthly

    b) Quarterly

    c) Every 4 months

    d) Continuously

    e) Yearly

Problem 5. The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0029t}$, where t represents the year, with $t = 6$ corresponding to 1996.

a) According to the model, is the population of Russia increasing or decreasing? Explain.

b) Find the population of Russia in 1998 and 2000.

Problem 6. Let Q represent a mass of carbon 14 $\left(^{14}C\right)$ (in grams), whose half-life is 5715 years. The quantity of carbon 14 present after t years is $Q = 10\left(\frac{1}{2}\right)^{t/5715}$.

a) Determine the initial quantity (when $t = 0$).

b) Determine the quantity present after 2000 years.

Homework: Read section 3.1, do #7, 17, 23, 29, 37, 51, 55, 63, 68